The Laplace Transform and The Laplace application in technique

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Summary:

The Laplace Transform is named after Pierre Simon Laplace, a French mathematician and astromer ( 1749-1827) as an important mathematic tool which has much application in many areas : mathematic, physic, mechanics … The sources of the application is the transformation of the differentland integral calculus of the functions to algebraic calculus of their imagine, so it makes solving linear ODEs, system of linear ODEs and related initial value problems, differtialand integral equations simpler. Basically, we study some of the application of the transform in engineering. The application in oscillation problems, the process which happen in electrical continuous time circuit, solving boundary problems.

INTRODUCITONS:

The Laplace transform is an integral transformation that allows conversion form integral calculus on the function f(t) of the real variable t to the operations calculate algebraic image F(s) of compels variables s, so that transformation helps to solve problems and problems first condition (equation, system of equations differential; system of the integral equation,…) about the new problem that contains the function that is algebraic equation in compels domain s. In spite of the system of the linear differential equations can be solved by the classical method, however, with the problems of signal that analysis technical is linear between input and output, can be showed by the differential equation linear distribution, the Laplace Transform has special application, on the other hand, when using the transform to solve equations with the initial conditions in the transform process. This transform is an effective tool for solving problems math with the initial condition that includes continuous circuits, mechanical system, especially when the input is discontinuous , periodic, or pulse function. Also, the Laplace transform also can provide calculation ways and graphical ways for analysis, synthesis and design system is easier. Consider the root functions f (t) of the real variable t, which are the continuous function interval with t; ∃ M>0,so and f(t)=0 when t<0

THE LAPLACE TRANSFORM:

Consider f(t) is a root function, then F(s)= is analytic function in a plane Res > 0 and is called imagine of f(t) through the Laplace transform, that can be showed by L

THE INVERTED LAPLACE TRANSFORM:

The transformation from F(s) becomes f(t) is called the inverted Laplace Transform f(t)=

THE APPLICATION OF LAPLACE TRANSFORM IN TECNIQUE

1. Application in oscillation problems:

a, Harmonic oscillation: Considering a point p that has a mass m and move straight in Ox, coordinate x(t) and attract a recovery force We can have mx’’ = -f1(t) ↔ x’’+x=0 with k/m=

b, Damped oscillation: Considering the external resistance force with and the first order rate with instantaneous velocity of a particle f2(t)=av(t). The mx’’+ax’(t)+kx=0 → x’’+2βx’+=0 with k/m=, α/m=2β

c, Forced oscillation without resistance force: Considering the case that point is still affected by cyclic external force f(t)= Hcos(. Hence we will have mx’’=- kx - αx’+ Hcos( → x’’+2βx’+ = Hcos(/m.

Example 1: Considering a system of springs (picture 1) , m1=m2=1, k1=k2=5/2. We choose the positive direction to the right and assume the position at t=0 is x1(0)=x1’(0),x2­­(0)=1, x2’(0)=0 and has the position x1(t) and x2(t) at time t.

Check that if we apply force f1(t)=0 and f2(t)=2(1-H(t-2)) with 2 masses, following Newton second law, we will have:

M1x1’’ = -(k1+k2)x1+k2x2+2(1-H(t-2))

M2 x2’’= k2 x1-(k2+k3)x2

→

With the Laplace transform we will obtain:

→ →

And the inverted Laplace:

x1(t)=11/24+(5/8)cos()+(1/6)cos(+(1/24)(-11+3cos(+ 8cos(.

3cos((.

The manual calculation is necessary to improve the thinking capacity, but the use of mathematical software is also useful in the learning process as well as research. In order to get quick results and survey in case of changing parameters such as mass of objects, elastic coefficient of springs, we perform calculations and function on some mathematical software such as Matlab , Maple (or use the built-in maple commands on Matlab)

Matlab function

[x1 x2]=funex1(m1,m2,k1,k2,k3,x10, x20,dx10,dx20)

d2x1 =

sym('diff(diff(x1(t),t),t)');

d2x2 =

sym('diff(diff(x2(t),t),t)');

x1 = sym('x1(t)');

x2= sym('x2(t)');

syms t s

H2=sym(Heaviside(t-2));

eq1=m1\*d2x1+(k1+k2)\*x1-k2\*x2- 2\*(1-H2);

eq2=m2\*d2x2-k2\*x1+(k2+k3)\*x2;

L1 = laplace(eq1,t,s);

L2 = laplace(eq2,t,s);

syms Lx1 Lx2

Nx1 =

subs(L1,{'x1(0)','D(x1)(0)','x 2(0)','D(x2)(0)'}, {x10,dx10,x20,dx20});

Nx2 = subs(L2,{'x1(0)','D(x1)(0)','x2(0)','D(x2)(0)'},{x10,dx10,x20,dx20});

Nx1=subs(Nx1,{'laplace(x1(t),t ,s)','laplace(x2(t),t,s)'},{Lx 1,Lx2});

Nx2=subs(Nx2,{'laplace(x1(t),t ,s)','laplace(x2(t),t,s)'},{Lx 1,Lx2});

[Lx1,Lx2] = solve(Nx1,Nx2,Lx1,Lx2);

x1= ilaplace(Lx1,s,t)

x2= ilaplace(Lx2,s,t) Picture

In the command prompt,

>>[x1 x2]=funex1(1,1,3,5/2,3,0,1,0,0)

>> t=[0:0.05:6];

>> x1=subs(x1,t);

>> x2=subs(x2,t);

>> plot(t,x1,t,x2)

The picture represents solution of the oscillation problem with time variable t on Matlab.

We also can use Maple:

>>maple('d:=diff(x1(t),t$2)=- 11\*x1(t)/2+5\*x2(t)/2+2\*(1- Heaviside(t2)),diff(x2(t),t$2)=5\*x1(t)/2- 11\*x2(t)/2;');

>> maple('f={x1(t),x2(t)}');

>>maple('dsolve({d,x1(0)=0,D(x 1)(0)=0,x2(0)=1,D(x2)(0)=0},{x 1(t),x2(t)}),method=laplace;')

Example 2

A oscillation in case without resistance force, is affected by a forced periodic force f(t)=H0cost and has a differential equation is x’’+ Hcos(t), H= , x(0)=a,x’(0)=b

With the Laplace Transform we have

→ X =

In the case :

X =

And with the inverted Laplace Transform:

→ x(t) = A cos (+ cos(t), A=

In case , x(t) = A cos (+ → A= ;

**Solution of boundary value problem**

Laplace transform method is also very useful in finding solutions of boundary problems, these problems often appear in the displacement theory of bending beams, or the problem of temperature distribution on a bar with a given heat source.

Consider a bar of length *l*, and consider a point K on the bar axis before deformation. After deformation, the point K changes to a point K '. The component v perpendicular to the bar axis is called the deflection of K. The differential equation for the vertical deflection y (x) of the bar is affected by the load W (x) per unit length at a given distance x. The original adjective coordinates on the axis of the bar are

Example 3:

A bar a length of l, and mass M and attached in 2 ends and has a load of Q in x=2/3l, determine the deflection?

Pictures

Because 2 ends are attached so the deflection and rotation angle at that points is equal 0.

Then,

0<x<l with y(0)=y’(0)=0, y(l)=y’(l)=0 (2)

Because Q is a Constance, by Laplace Transform:

* (3)

Derivativation, changing the boundary condition at *l:*

Then (3) becomes:

+ 0<x<1.

Using Matlab:

>>syms EI M Q l;

>>maple('d:=EI\*diff(y(x),x$4)= M/l+Q\*Dirac(x-2\*l/3);');

>>maple('dsolve({d,y(0)=0,y(l) =0,D(y)(0)=0,D(y)(l)=0},y(x)), method=laplace;')

**Application in circuit:**

Laplace transforms are an effective tool when studying transients in electronic circuits when they are suddenly applied by short-term electrical impulses or during switching. That process is called the response of an electronic circuit to a step pulse at times t 0.

According to Ohm's law, the amount of potential voltage when the current through the resistor R is vr(t)=Ri(t), inductor L is vL(t)= L and conductor C is vc(t)= , i(t)=

Picture

Apply Kirchoff’s Law”

L + Ri(t)+

Apply Laplace we will have RI(s)+LsI(s)+

Let’s assume: Z(s)= R+Ls+is the characteristic impedance of the circuit so U(s)=Z(s)I(s) is the expression of Ohm's law in terms of the operator of the circuit.

**Transfer function of a linear system**

The response of a system to a defined impact at the entrance depends on the characteristics of the system, which is contained in a quantity called the transfer function of the system. The transfer function H (s) of the system is determined by the ratio of the Laplace transform of the output signal on the Laplace transform on the input signal H(s) =.

Example 4:

Given the circuit shown in the figure, find the transmission matrix for the outputs y1 and y2 corresponding to the variables u. Calculate the response when u=H(t) with R1=2, R2=1.5, C=3/7,L=1/7

Picture

Apply Kirchoff

e=R1(x2+Cx2’)+x1+R2Cx1’

* =

e=R1(x2+Cx1’)+x1+Lx1’

with output variable

=

Then we put the numbers to the matrix

* =
* =

So the matrix transform will be

G(s)=C x(0)=0 so Y(s)=G(s)(1/s), with the inverted Laplace

* y(t) = =
* Using Matlab. We create the following function, the result is a transmission matrix and y

function[y G]=funex2(R1, R2, C, L, u)

A=[-1/(C\*(R1+R2)) - R1/(C\*(R1+R2));R1/(L\*(R1+R2)) -R1\*R2/(L\*(R1+R2))];

C1=[0 1;-1/(R1+R2) - R1/(R1+R2)];

b=[1/(C\*(R1+R2));R2/(L\*(R1+R2) )];

d=[0;1/(R1+R2)];

syms s

G=C1\*(s\*eye(2)-A)^(-1)\*b+d

U=laplace(u);

Y=G\*U;

y=ilaplace(Y);

In the command prompt :

>> [y G]=funex2(2, 1.5, 3/7, 1/7, sym('Heaviside(t)')) t=[0:0.05:5];

>> y1=subs(y(1,1),t);

>> y2=subs(y(2,1),t);

>> plot(t,y1,t,y2)

Picture